

# Laplace Transforms Formulae

$f(t)$ or $\mathcal{L}^{-1}(F(s))$	$\mathcal{L}(f(t))$ or $F(s)$
$t^n$	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s-a}$
$\sin kt$	$\frac{k}{s^2+k^2}$
$\cos kt$	$\frac{s}{s^2+k^2}$

## Translation Theorems

$$\begin{aligned}\mathcal{L}(e^{at}f(t)) &= F(s-a) \\ \mathcal{L}(f(t-a)\mathcal{U}(t-a)) &= e^{-as}F(s) \\ \mathcal{L}(f(t)\mathcal{U}(t-a)) &= e^{-as}\mathcal{L}(f(t+a))\end{aligned}$$

## Derivatives and Integrals

$$\begin{aligned}\mathcal{L}(f'(t)) &= s\mathcal{L}(f(t)) - f(0) \\ \mathcal{L}(f''(t)) &= s^2\mathcal{L}(f(t)) - sf(0) - f'(0) \\ \mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} &= \frac{F(s)}{s}\end{aligned}$$

## Additional Operational Properties

$$\begin{aligned}\mathcal{L}(t^n f(t)) &= (-1)^n \frac{d^n}{ds^n} F(s) \\ \mathcal{L}(\delta(t-a)) &= e^{-as} \\ \mathcal{L}(\mathcal{U}(t-a)) &= \frac{e^{-as}}{s}\end{aligned}$$

Define the *Convolution* as:

$$\begin{aligned}f * g &= \int_0^t f(\tau)g(t-\tau)d\tau \\ \mathcal{L}(f * g) &= \mathcal{L}(f) \cdot \mathcal{L}(g)\end{aligned}$$

For a periodic function  $f$  with period  $T$ :

$$\mathcal{L}(f(t)) = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$